

## Workshop Alpbach 2022

Hotel Böglerhof, Alpbach/Austria  
June 26 - July 1, 2022

Organizers: J. Ayoub, C. Fuchs, Ph. Habegger, R. Pink, G. Wüstholz, S. Zerbes

Supported by: ETH Zurich, University of Zurich, University of Salzburg, Austrian Science Fund (FWF): I4406

This, the 15th in a series of workshops in Alpbach, will feature minicourses given by world class researchers and invited talks by younger researchers, covering a spectrum of in arithmetic geometry related to Galois representations and heights. The emphasis includes not only deep theoretical developments, but also applications of a more concrete/computational nature. Minicourses presenting a broad overview of these topics, delivered by top international experts, will be complemented by invited talks highlighting recent progress.

*Minicourses given by:*

Ana Caraiani (Imperial C. London)  
Philipp Habegger (U. Basel)  
Urs Hartl (U. Münster)

*Talks given by:*

Ziyang Gao (U. Hannover)  
Lars Kühne (U. Hannover)  
Zhenlin Ran (U. of Newcastle)  
Robert Wilms (U. Basel)

## Program

All lectures take place in the seminar room at Hotel Böglerhof.

### Monday, June 27, 2022:

- 09:00 - 10:30: **Philipp Habegger** (U. Basel)  
*The Schinzel-Zassenhaus Conjecture and Arithmetic Dynamics. I*
- 11:00 - 12:30: **Ana Caraiani** (Imperial C. London)  
*Cohomology of Shimura varieties. I*
- 13:45 - 15:15: **Zhenlin Ran** (U. of Newcastle)  
*Heights and singular moduli of Drinfeld modules*

### Tuesday, June 28, 2022:

- 09:00 - 10:30: **Ana Caraiani** (Imperial C. London)  
*Cohomology of Shimura varieties. II*
- 11:00 - 12:30: **Philipp Habegger** (U. Basel)  
*The Schinzel-Zassenhaus Conjecture and Arithmetic Dynamics. II*
- 13:45 - 15:15: **Ziyang Gao** (U. Hannover)  
*Torsion points in families of abelian varieties*

### Wednesday, June 29, 2022:

- 09:00 - 10:30: **Philipp Habegger** (U. Basel)  
*The Schinzel-Zassenhaus Conjecture and Arithmetic Dynamics. III*
- 11:00 - 12:30: **Ana Caraiani** (Imperial C. London)  
*Cohomology of Shimura varieties. III*
- 13:45 - 15:15: **Urs Hartl** (U. Münster)  
*Shtukas for function fields. I*

**Thursday, June 30, 2022:**

09:00 - 10:30: **Urs Hartl** (U. Münster)  
*Shtukas for function fields. II*

11:00 - 12:30: **Robert Wilms** (U. Basel)  
*On the irreducibility of arithmetic divisors*

**Friday, July 1, 2022:**

09:00 - 10:30: **Urs Hartl** (U. Münster)  
*Shtukas for function fields. III*

11:00 - 12:30: **Lars Kühne** (U. Hannover)  
*The Relative Bogomolov Conjecture for Fibered Products of Elliptic Families*

## Abstracts

**Ana Cariani** (Imperial C. London)

Title: *Cohomology of Shimura varieties*

Abstract: I will survey some vanishing conjectures for the cohomology of Shimura varieties with torsion coefficients under both local and global conditions. I will then describe the  $p$ -adic geometry of Shimura varieties and the associated Hodge-Tate period morphism, and I will explain how this can be used to make progress on these conjectures. Finally, I will mention some applications of these results to (potential) modularity over CM fields.

**Philipp Habegger** (U. Basel)

Title: *The Schinzel-Zassenhaus Conjecture and Arithmetic Dynamics*

Abstract: The Schinzel-Zassenhaus Conjecture states that a non-zero algebraic integer of degree  $d$  that is not a root of unity has at least one conjugate with absolute value greater than  $1 + c/d$  where  $c > 0$  is an absolute constant. This conjecture was proved in 2019 in breakthrough by Vesselin Dimitrov. In the context of arithmetic dynamics there is an analog due to Harry Schmidt and myself. This analog covers as a special case the dynamical system determined by  $T^2 - 1$ . The plan is to give an overview of Dimitrov's proof. Then I will cover some fundamental concepts in arithmetic dynamics. These include the local canonical height and Julia sets in the archimedean setting and the role they play in the proof.

**Urs Hartl** (U. Münster)

Title: *Shtukas for function fields*

Abstract: Shtukas over function fields were discovered by Drinfeld 35 years ago as analogs of elliptic curves and abelian varieties. Moduli spaces of shtukas are function field analogs of Shimura varieties and were used to prove the Langlands correspondence over global function fields for  $GL_2$  by Drinfeld and for  $GL_n$  by L. Lafforgue. The group  $GL_n$  can be replaced by a linear algebraic group  $G$  to obtain global  $G$ -shtukas, whose moduli spaces were used for many further important results on the Langlands conjectures and in arithmetic geometry. In the theory of Shimura varieties and abelian varieties,  $p$ -divisible groups are inevitable tools. In a similar way global  $G$ -shtukas give rise to local  $G$ -shtukas. In this minicourse we define global shtukas and develop their arithmetic theory. We present their moduli spaces and their importance for the Langlands program. We explain the relation between global and local  $G$ -shtukas. Inspired by the power of local  $G$ -shtukas, Scholze developed a theory of shtukas over  $p$ -adic fields, which we discuss in the end.

**Zhenlin Ran** (U. of New Castle)

Title: Heights and singular moduli of Drinfeld modules

Abstract: Drinfeld modules are usually regarded as the analogue of elliptic curves in function field world. In an attempt to prove Tate conjecture in the context of Drinfeld modules in 1993, Taguchi introduced his so-called differential height of Drinfeld modules which is now usually called Taguchi height. Taguchi height is the best analogue of Faltings height of abelian varieties in the context of Drinfeld modules. In this talk, I will introduce the minimal model and Taguchi height of Drinfeld modules, and talk about some results of calculating the variation of Taguchi heights of isogenous CM Drinfeld modules. In particular, as an application, I will talk about an analogous result in the context of rank 2 Drinfeld modules of a theorem of singular moduli from Habegger. If time permits, I will explain our way of controlling Galois orbits near the zeros of  $j$ -function in function field.

**Ziyang Gao** (U. Hannover)

Title: *Torsion points in families of abelian varieties*

Abstract: Given an abelian scheme defined over  $\overline{\mathbb{Q}}$  and an irreducible subvariety  $X$  which dominates the base, the Relative Manin-Mumford Conjecture (proposed by Zannier) predicts how torsion points in closed fibers lie on  $X$ . The conjecture says that if such torsion points are Zariski dense in  $X$ , then the dimension of  $X$  is at least the relative dimension of the abelian scheme, unless  $X$  is contained in a proper subgroup scheme. In this talk, I will present a proof of this conjecture. As a consequence this gives a new proof of the Uniform Manin-Mumford Conjecture for curves (recently proved by Kühne) without using equidistribution. This is joint work with Philipp Habegger.

**Robert Wilms** (U. Basel)

Title: *On the irreducibility of arithmetic divisors*

Abstract: In Arakelov theory one studies an intersection theory over a hypothetical completion of the spectrum of integers. Thus, an arithmetic divisor comes with the additional datum of a green function, which has logarithmic poles exactly at the support of the divisor. In classical geometry it is often useful to work with irreducible divisors. Unfortunately, a (horizontal) arithmetic divisor is never irreducible in the naive sense, as the green function can always be decomposed into the sum of another green function of the same divisor and a continuous function. In this talk I will introduce a new notion of epsilon-irreducibility for arithmetic divisors meaning that the integral of the green function is bounded by some epsilon. The main result is that any arithmetically ample hermitian line bundle  $L$  is  $\epsilon$ -arithmetically irreducibly representable, meaning that for every epsilon some tensor power

of  $L$  is representable by an epsilon-irreducible arithmetic divisor. As a main step of the proof I will present a new equidistribution result for the global sections of arithmetically ample hermitian line bundles. As an application, I will discuss an inequality of arithmetic intersection numbers of a great generality.

**Lars Kühne** (U. Hannover)

Title: *The Relative Bogomolov Conjecture for Fibered Products of Elliptic Families*

Abstract: I will talk about the deduction of the Bogomolov conjecture for non-degenerate subvarieties in fibered products of elliptic families from my recent theorem on equidistribution in families of abelian varieties. This generalizes results of DeMarco and Mavraki and improves certain results of Manin-Mumford type proven by Masser and Zannier to results of Bogomolov type.