

Workshop “Alpbach 2018: Galois Representations and Heights”

Hotel Böglerhof, Alpbach/Austria
July 1-6, 2018

Organizers: J. Ayoub, C. Fuchs, R. Pink, G. Wüstholz

Supported by: ETH Zurich, University of Zurich, University of Salzburg

This, the 12th in a series of workshops in Alpbach, will feature minicourses given by world class researchers and invited talks by younger researchers, covering a spectrum of in arithmetic geometry related to Galois representations and heights. The emphasis includes not only deep theoretical developments, but also applications of a more concrete/computational nature. Minicourses presenting a broad overview of these topics, delivered by top international experts, will be complemented by invited talks highlighting recent progress.

Minicourses given by:

Jennifer Balakrishnan (Boston U.)

Jack Thorne (U. Cambridge)

Keerthi Madapusi Pera (Boston College)

Invited Talks:

Olivier Benoist (Strasbourg)

Yohan Brunebarbe (U. Zurich)

Sergey Gorchinskiy (Steklov Math. Inst., NRU HSE, Moscow)

Ernst Kani (Queen’s U. at Kingston)

Program

All lectures take place in the seminar room at Hotel Böglhof.

Sunday, July 1, 2018:

- 17:00 - 17:15: Opening
Welcome words and organizational matters
- 17:15 - 18:45: **Ernst Kani** (Queen's U. at Kingston)
Moduli problems associated to isomorphisms of elliptic Galois representations

Monday, July 2, 2018:

- 09:00 - 10:30: **Jack Thorne** (U. Cambridge)
Modularity over imaginary quadratic fields. part I
- 11:00 - 12:30: **Jennifer Balakrishnan** (Boston U.)
Quadratic Chabauty. part I
- 13:45 - 15:15: **Keerthi Madapusi Pera** (Boston College)
Heights of abelian varieties with complex multiplication. part I

Tuesday, July 3, 2018:

- 09:00 - 10:30: **Jack Thorne** (U. Cambridge)
Modularity over imaginary quadratic fields. part II
- 11:00 - 12:30: **Jennifer Balakrishnan** (Boston U.)
Quadratic Chabauty. part II
- 13:45 - 14:45: **Olivier Benoist** (Strasbourg)
A real period-index theorem
- 15:00 - 16:00: **Yohan Brunebarbe** (U. Zurich)
Hyperbolicity of moduli spaces of Calabi-Yau varieties with level structure

Wednesday, July 4, 2018:

09:00 - 10:30: **Keerthi Madapusi Pera** (Boston College)
Heights of abelian varieties with complex multiplication. part II

11:00 - 12:30: **Jennifer Balakrishnan** (Boston U.)
Quadratic Chabauty. part III

Thursday, July 5, 2018:

09:00 - 10:30: **Jack Thorne** (U. Cambridge)
Modularity over imaginary quadratic fields. part III

11:00 - 12:30: **Keerthi Madapusi Pera** (Boston College)
Heights of abelian varieties with complex multiplication. part III

13:45 - 14:45: **Sergey Gorchinskiy** (Steklov Math. Inst., NRU HSE, Moscow)
Relative Milnor K -groups and differential forms of split nilpotent extensions

Friday, July 6, 2018:

09:00 - 10:30: **Keerthi Madapusi Pera** (Boston College)
Heights of abelian varieties with complex multiplication. part IV

11:00 - 12:30: **Jack Thorne** (U. Cambridge)
Modularity over imaginary quadratic fields. part IV

Abstracts

Jennifer Balakrishnan (Boston U.)

Title: *Quadratic Chabauty*

Abstract: I will discuss various results regarding explicit methods for “quadratic Chabauty,” which can be used to find rational or integral points on certain curves of genus at least 2. In particular, I will describe how to compute p -adic iterated integrals and p -adic heights and some recent examples where we’ve used these techniques to compute rational points on curves.

Jack Thorne (U. Cambridge)

Title: *Modularity over imaginary quadratic fields*

Abstract: In 1995, Wiles (and Taylor–Wiles) proved Fermat’s Last Theorem, by proving the first modularity lifting theorems for two-dimensional Galois representations of the absolute Galois group of \mathbb{Q} . 20 years later modularity of 2-dimensional Galois representations over totally real fields is a mature subject. Stand-out results are the proof of Serre’s conjecture by Khare–Wintenberger and Taylor’s work on potential modularity over totally real fields. Until quite recently, going beyond the case of a totally real field seemed like an insurmountable problem. Calegari–Geraghty laid the first steps, giving a conjectural generalization of the Taylor–Wiles method based upon at that time unknown properties of the cohomology of arithmetic groups (like $\mathrm{SL}_2(O_K)$, where K is an imaginary quadratic field). Work in progress by a large group of authors has led to the possibility of proving unconditional theorems. I will discuss some of the context surrounding this work.

Keerthi Madapusi Pera (Boston College)

Title: *Heights of abelian varieties with complex multiplication*

Abstract: In the 90s, P. Colmez made a striking conjecture relating the heights of CM abelian varieties to the values at 0 of the derivatives of Artin L-functions. Recently, there has been work by several people proving this conjecture ‘in the average’. Among other things this has led to the completion of the proof of the Andre-Oort conjecture by Pila-Tsimerman. This course will recount one of the methods used to prove Colmez’s conjecture, which employs methods from S. Kudla’s program relating intersection numbers on orthogonal Shimura varieties with Fourier coefficients of the derivatives of certain Eisenstein series.

Olivier Benoist (Strasbourg)

Title: *A real period-index theorem*

Abstract: De Jong has proven that the period and the index of a class in the Brauer group of the function field of a complex surface coincide. We prove

the same statement for classes in the Brauer group of the function field of a real surface that are trivial in restriction to the real points of the surface. As a consequence, we show that the u-invariant of the function field of a real surface is equal to 4. In this talk, after explaining and motivating these statements, we will sketch the proof, that relies on Hodge theory.

Yohan Brunebarbe (U. Zurich)

Title: *Hyperbolicity of moduli spaces of Calabi-Yau varieties with level structure*

Abstract: Moduli stacks of polarized complex varieties whose canonical bundle is trivial are known to enjoy many hyperbolicity properties. In this talk, I will explain how adding level structure to the picture yields much stronger results.

Sergey Gorchinskiy (Steklov Math. Inst., NRU HSE, Moscow)

Title: *Relative Milnor K -groups and differential forms of split nilpotent extensions*

Abstract: The talk is based on common works with Dimitrii Tyurin and with Denis Osipov. We prove a version of the famous Goodwillies theorem with algebraic K -groups being replaced by Milnor K -groups. Namely, given a commutative ring with a nilpotent ideal such that the quotient splits, we construct an isomorphism between the relative Milnor K -group of degree $n + 1$ and the quotient of the relative module of differential forms of degree n over the de Rham differential of the analogous module of degree $n - 1$. For this we also assume that the rings have enough invertible elements in a sense. This theorem is in line with previously known numerous results from algebraic K -theory. Nevertheless, unlike them, our proof is based only on the Steinberg relation and on explicit tricks with symbols in Milnor K -groups.

Ernst Kani (Queen's U. at Kingston) Title: *Moduli problems associated to isomorphisms of elliptic Galois representations*

Abstract: The study of isomorphisms between mod N Galois representations attached to elliptic curves over a number field K leads to important conjectures due Frey, Mazur and others. These conjectures are closely connected with other Diophantine conjectures such as the asymptotic Fermat conjecture, etc. If we fix the integer N , then such isomorphisms are classified by an algebraic surface \mathbb{Z}_N/\mathbb{Q} , which consists of $\phi(N)$ irreducible components. In my talk I will discuss some properties of this surface. In addition, I will explain how one component of this surface is related to the Hurwitz space of genus 2 covers of degree N of elliptic curves, and also to the Humbert surface H_{N^2} of discriminant N^2 .