SALZBURG MATHEMATICS COLLOQUIUM

Winter 2014/2015

Paolo Marcellini (Florence)

„Some explicit solutions to a system of implicit partial differential equations: rigid maps and origami“

October 23, 2014

Abstract: A rigid map u is a Lipschitz-continuous map with the property that at every point where u is differentiable its gradient is an orthogonal matrix. We introduce Lipschitz-continuous local isometric immersions and propose an approach to the analytic theory of origami (i.e. piecewise C^1 rigid maps plus a condition which excludes self intersection). We characterize the singular set of u and use this characterization to explicity solve a class of Dirichlet problems associated to some partial differential systems of implicit type. For more information see the extended abstract at the web page mentioned below.

Thursday, **15:15-16:00**
Seminarraum II, 1. Stock

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Salzburg Mathematics Colloquium

Winter 2014/2015

Paolo Marcellini (Florence)

“Some explicit solutions to a system of implicit partial differential equations: rigid maps and origami”

October 9, 2014

Extended Abstract:

A *rigid map* $u\colon \Omega \subset \mathbb{R}^n \to \mathbb{R}^m$ is a *Lipschitz-continuous map* with the property that at every $x \in \Omega$ where u is differentiable then its gradient $Du(x)$ is an *orthogonal* $m \times n$ *matrix; i.e.,* $Du(x) \in O(n)$. If Ω is convex, then u is globally a *short map*, in the sense that $|u(x)-u(y)| \leq |x-y|$ for every $x, y \in \Omega$; while locally, around any point of continuity of the gradient, u is an *isometry*. Our motivation to introduce Lipschitz-continuous local isometric immersions (versus maps of class C^1) is based on the possibility of solving *Dirichlet problems*; i.e., we can impose boundary conditions.

We also propose an approach to the **analytical theory of origami**, the ancient Japanese art of paper folding. An *origami* is a piecewise C^1 rigid map $u\colon \Omega \subset \mathbb{R}^2 \to \mathbb{R}^3$ (plus a condition which exclude self intersections). If the image of the map $u(\Omega)$ is in $\mathbb{R}^2$ we say that u is a *flat origami*. In this case (and in general when $m = n$) we are able to describe the singular set Σ_u of the gradient Du of a piecewise C^1 rigid map: it turns out to be the boundary of the union of convex disjoint polyhedra, and some facet and edge conditions *(angle condition)* are satisfied. We show that these necessary conditions are also sufficient to recover a given singular set; i.e., that every polyhedral set Σ which satisfies the *angle condition* is in fact the *singular set* Σ_u of a map u, which is uniquely determined once we fix the value $u(x_0) \in \mathbb{R}^n$ and the gradient $Du(x_0) \in O(n)$ at a single point $x_0 \in \Omega \setminus \Sigma$. We use this characterization to explicitly solve a class of *Dirichlet problems* associated to some *partial differential systems* of *implicit type*.

This is a research in collaboration with Bernard Dacorogna (EPFL, Lausanne, Switzerland) and Emanuele Paolini (University of Firenze, Italy).

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