On the distribution of polynomials with real and integer coefficients

Attila Pethő, University of Debrecen

October 13, 2012

My talk is based on joint work with Shigeki Akiyama, which we did in Niigata during this snowily winter, where I worked with a long time research fellowship of JSPS.

Let $\mathcal{E}_d \subset \mathbb{R}^d$ denote the set of coefficients of monic polynomials of degree d with roots inside or on the unit circle. This is a bounded set, which can be divided naturally into $\lfloor d/2 \rfloor + 1$ subsets according the signature of the polynomial, i.e., according the number of its real roots. Let $\mathcal{E}_d^{(r,s)} \subset \mathcal{E}_d$ denote the set with signature $(r,s), r+2s=d$. In the talk we answer questions like: what is the probability that picking a point of $\mathcal{E}_d$ the corresponding polynomial is totally real? More generally what is the probability that picking a point of $\mathcal{E}_d$ the corresponding polynomial has signature (r,s)? Are these probabilities similar?

For a monic polynomial P with integer coefficients denote $|\overline{P}|$ its, in modulus largest root. In the second part of the talk we investigate the function $N_d^{(r,s)}(B)$ and $I_d^{(r,s)}(B)$, which denote the number of monic polynomials P with integer coefficients of degree d, with signature (r,s) and $|\overline{P}| < B$. We prove asymptotic formulae with quite good error term for these and related functions. Our results give some explanation for the empirical fact that picking a polynomial with integer coefficients we obtain nearly always an irreducible one.

You can download the manuscripts at the ULR:
http://www.inf.unideb.hu/ pethoe/cikkek/realandint_poly_v5.pdf and
http://www.inf.unideb.hu/ pethoe/cikkek/int_poly_v4.pdf