# A uniform open image theorem for $p$-adic representations of etale fundamental groups of curves (joint work with Akio Tamagawa - R.I.M.S.)

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Let $k$ be a finitely generated field of characteristic 0, $X$ a smooth, separated, geometrically connected curve over $k$ with generic point $\eta$. Fix a prime $p$. A representation $\rho : \pi_1(X) \to \mathrm{GL}_d(\mathbb{Z}_p)$ is said to be strictly geometrically nonabelian if $\rho(\pi_1(X_{\overline{k}}))$ has finite abelianization. Typical examples of such representations are those arising from the action of $\pi_1(X)$ on the generic Tate module $T_p(A_\eta)$ of an abelian scheme $A$ over $X$. Let $G$ denote the image of $\rho$. Any $k$ -rational point $x$ on $X$ induces a splitting $x : \Gamma_k \to \pi_1(X)$ of the canonical restriction epimorphism $\pi_1(X) \to \Gamma_k$ so one can define the closed subgroup $G_x := \rho \circ x(\Gamma_k) \subset G$. The main result I am going to discuss is the following uniform open image theorem. Under the above assumptions, for any strictly geometrically nonabelian representation $\rho : \pi_1(X) \to \mathrm{GL}_d(\mathbb{Z}_p)$ the set $X_\rho$ of all $x \in X(k)$ such that $G_x$ is not open in $G$ is finite and there exists an integer $B_\rho \geq 1$ such that $[G : G_x] \leq B_\rho,\ x \in X(k) \setminus X_\rho$.

Applied to the action of $\pi_1(X)$ on the generic Tate module $T_p(A_\eta)$ of an abelian scheme $A$ over $X$, this result yields a uniform open image theorem and a uniform boundedness theorem of the (twisted) $p$-primary torsion for families of higher dimensional abelian varieties parametrized by curves.

If time allows, I will also try and indicate how to prove - in the case of number fields - a strong variant of our result: for any strictly geometrically nonabelian representation $\rho : \pi_1(X) \to \mathrm{GL}_d(\mathbb{Z}_p)$ and for any integer $d \geq 1$ the set $X_{\rho,d}$ of all closed points $x \in X$ such that $[k(x) : k] \leq d$ and $G_x$ is not open in $G$ is finite and there exists an integer $B_{\rho,d} \geq 1$ such that $[G : G_x] \leq B_{\rho,d},\ x \notin X_{\rho,d}$ with $[k(x) : k] \leq d$.