Workshop "Arithmetic Algebraic Geometry"

Hotel Böglerhof, Alpbach/Austria
July 9-14, 2016

Organizers: J. Ayoub, C. Fuchs, A. Kresch, R. Pink, G. Wüstholz
Supported by: ETH Zurich, University of Zurich, University of Salzburg

This, the 11th in a series of workshops in Alpbach, will feature minicourses and invited talks on topics covering a spectrum of modern arithmetic geometry. Minicourses presenting a broad overview of these topics, delivered by top international experts, will be complemented by invited talks highlighting recent progress.

Minicourses given by:
Brian Conrad (Stanford)
Samir Siksek (Warwick)
Wei Zhang (Columbia)

Invited Talks:
Javier Fresán (ETH Zurich)
Ariyan Javanpeykar (Mainz)
Dimitar Jetchev (EPFL)
Yunqing Tang (Princeton)

Program

All lectures take place in the seminar room at Hotel Böglerhof.

Sunday, July 9, 2017:

17:00 - 17:15: Opening
Welcome words and organizational matters

17:15 - 18:45: **Dimitar Jetchev** (EPFL)
Arithmetic properties of special cycles on unitary Shimura varieties

Monday, July 10, 2017:

09:00 - 10:30: **Wei Zhang** (Columbia)
Shtukas and higher Gross-Zagier formula. part I

11:00 - 12:30: **Samir Siksek** (Warwick)
Galois representations of elliptic curves and Diophantine applications. part I

13:45 - 15:15: **Brian Conrad** (Stanford)
Pseudo-reductive groups: structure theory and arithmetic applications. part I

Tuesday, July 11, 2017:

09:00 - 10:30: **Samir Siksek** (Warwick)
Galois representations of elliptic curves and Diophantine applications. part II

11:00 - 12:30: **Brian Conrad** (Stanford)
Pseudo-reductive groups: structure theory and arithmetic applications. part II

13:45 - 14:45: **Javier Fresán** (ETH Zurich)
Exponential motives

15:00 - 16:00: **Ariyan Javanpeykar** (Mainz)
Arithmetic, algebraic, and analytic hyperbolicity

18:00 - 19:00: **Yunqing Tang** (Princeton)
Exceptional splitting of reductions of abelian surfaces with real multiplication

Wednesday, July 12, 2017:

09:00 - 10:30: **Samir Siksek** (Warwick)
Galois representations of elliptic curves and Diophantine applications. part III

11:00 - 12:30: **Wei Zhang** (Columbia)
Shtukas and higher Gross-Zagier formula. part II

Thursday, July 13, 2017:

09:00 - 10:30: **Wei Zhang** (Columbia)
Shtukas and higher Gross-Zagier formula. part III

11:00 - 12:30: **Brian Conrad** (Stanford)
Pseudo-reductive groups: structure theory and arithmetic applications. part III

13:45 - 15:15: **Samir Siksek** (Warwick)
Galois representations of elliptic curves and Diophantine applications. part IV

Friday, July 14, 2017:

09:00 - 10:30: **Brian Conrad** (Stanford)
Pseudo-reductive groups: structure theory and arithmetic applications. part IV

11:00 - 12:30: **Wei Zhang** (Columbia)
Shtukas and higher Gross-Zagier formula. part IV

Abstracts

Brian Conrad (Stanford)
Title: *Pseudo-reductive groups: structure theory and arithmetic applications*
Abstract: We give an overview of examples, structure, and arithmetic applications of pseudo-reductive groups (developed in collaboration with O. Gabber and G. Prasad). After some preliminary motivation for the study of pseudo-reductivity, we will see that the internal structure theory of such groups has many similarities to the reductive case (e.g., root groups, relative root systems, open cell, and Bruhat decomposition) but often requires rather different proofs (e.g., root groups must be developed in an entirely different manner, and SL(2) does not play the same key role as in the reductive case but it remains important). Such groups can also exhibit features that deviate from the reductive case (e.g., nontrivial central etale p-torsion in characteristic p, and loss of pseudo-reductivity under smooth quotients). The key to the usefulness of the theory (and the ingredient missed in the early work of Borel and Tits on this topic) is the "standard construction". The main aim of the lectures is to explain the proof of exhaustiveness of the standard construction over fields not of characteristics 2 or 3, as well as the additional possibilities that exist in these small characteristics. This provides a very satisfactory classification "modulo the commutative case", and we will illustrate some ways in which the ubiquity of standardness can be applied (and how one handles non-standard examples in small characteristic).

Samir Siksek (Warwick)
Title: *Galois representations of elliptic curves and Diophantine applications*
Abstract: The ideas that led to the proof of Fermat's Last Theorem, due to Frey, Serre, Ribet and Wiles, have found fruitful applications in many other Diophantine contexts. The strategy rarely solves Diophantine equations completely, but can be successful when integrated with other methods such as Baker's theory or even classical analytic number theory. We survey the capabilities of this "modular approach", and take a peek at the emerging Diophantine landscape around modularity theorems over number fields.

Wei Zhang (Columbia)
Title: *Shtukas and higher Gross-Zagier formula*
Abstract: We study automorphic L-functions for GL(2) over a global field (e.g., those attached to elliptic curves). In the case of the rational number field, the Gross-Zagier formula expresses the first derivative in terms of the height of Heegner point on a modular curve. In the case of function fields, in a joint work with Zhiwei Yun, we found a family of algebraic cycles on the moduli of Drinfeld Shtukas, and we prove that their intersection num-

bers give higher order derivatives of L-function. The lectures will review the backgrounds and we will present the main ingredients in the proof. If time permits, we also discuss several constructions of algebraic cycles on Shimura varieties over number fields.

Javier Fresán (ETH Zurich)
Title: *Exponential motives*
Abstract: Exponential periods form a class of complex numbers containing the special values of the gamma and the Bessel functions, the Euler-Mascheroni constant and other interesting numbers which are not expected to be periods in the usual sense of algebraic geometry. However, one can interpret them as coefficients of the comparison isomorphism between two cohomology theories associated to varieties together with a regular function: the de Rham cohomology of a connection with irregular singularities and the so-called "rapid decay" cohomology. I will explain how this point of view allows one to construct - following ideas of Katz, Kontsevich and Nori - a Tannakian category of exponential motives over a subfield of the complex numbers and to produce Galois groups which conjecturally govern all algebraic relations among these numbers. This is a joint work with Peter Jossen.

Ariyan Javanpeykar (Mainz)
Title: *Arithmetic, algebraic, and analytic hyperbolicity*
Abstract: The Lang-Vojta conjecture predicts that a complex algebraic variety is algebraically hyperbolic if and only if it is analytically hyperbolic if and only if it is arithmetically hyperbolic. This conjecture is known for the moduli of polarized abelian varieties by work of Borel, Faltings, and Zuo. In this talk, I will explain this conjecture, and its arithmetic consequences for the moduli of smooth projective hypersurfaces and Calabi-Yau varieties. Finally, I will discuss the conjecture for the moduli space of canonically polarized varieties. This is joint work with Daniel Loughran.

Dimitar Jetchev (EPFL)
Title: *Arithmetic properties of special cycles on unitary Shimura varieties*
Abstract: We discuss recent progress on an ongoing project for constructing an Euler system from special cycles on certain unitary Shimura varieties. We explain the arithmetic properties of the special cycles and explain how these properties lead to norm-compatibility relations similar to the relations satisfied by classical cyclotomic units and Heegner points. Our results will cover both split and inert primes. These constructions are useful in the study of Selmer groups of Galois representations attached to certain automorphic forms on the corresponding unitary groups and higher-dimensional analogues of the Birch and Swinnerton-Dyer conjecture.

Yunqing Tang (Princeton)

Title: *Exceptional splitting of reductions of abelian surfaces with real multiplication*